ANALYSIS OF GRANULAR FLOW BY CELLULAR AUTOMATA

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Abstract: The objective of this study is to propose a new method to simulate the granular flow with Cellular Automata (CA). First, it is demonstrated through the preparatory numerical experiment that Moore's neighborhood is reasonable as a basic rule to realize the effect of gravity. However, the rule in which only the effect of gravity is taken into consideration is not satisfactory. Then, the effect of friction between the grains and between the grain and wall is combined into the rule. Rice grains are used in the experiment of granular flow. The results of experiment of granular flow are compared with the simulation by the method proposed in this study.

Keywords **:** Cellular Automata, grain, flow, friction

1. INTRODUCTION

There are a plenty of complex phenomena. The study to simulate the behavior of such the complex phenomena considering it as continuity of local action is being continued. Cellular Automata (CA) has attracted interest recently as one of the methods. However, there are not many examples of application of CA and the name is not known widely in the field of civil engineering.

CA was proposed by John Von Neumann from the forties to the fifties of the twentieth century[1)]. A complex system is modeled by a set of cells. Cells are allocated in grid pattern and the state of each cell is determined with the state of the neighboring cells. The behavior of whole region can be simulated by activating the performance simultaneously. CA shows the best performance to the problem in which the behavior can not be expressed by differential equations.

Not a few but many complex systems have been analyzed by CA since CA made it possible to express phenomena by simple localized interaction of cells. It was applied in the wide range of field such as bioengineering, genetics, diffusion, crystal growth and net of roads[2)].

In civil engineering, various complex phenomena are to be analyzed. The problem of flow of fresh concrete is one of those. But the method has not been developed to simulate it accurately. As a preliminary stage, we pay attention to the analysis of granular flow and discuss how accurately it can explain the phenomenon, aiming to apply CA to the problem of flow of fresh concrete in the near future.

Discrete Element Method (DEM) [3)] is a typical method of analysis of granular flow. In this method, a grain is modeled with a mass, spring and damper. And equation of motion is written down to each grain, which is solved using the finite difference in a numerical analysis. It can explain the phenomenon more accurately compared with the analysis using continuum model. However, it has the difficulty in the determination of values of parameters. It also has the problem of taking a great deal of calculating time, since it needs very small time interval.

CA can be also applied to the analysis of granular flow. It has the ability to express the complex phenomenon, which develops after a time, by the localized interaction. The feature of CA is that the rule of local neighborhood and state transition can be given originally. Though several researchers have applied CA to the problem of granular flow[4-8)], the effect of horizontal force to grains and the friction is not necessarily properly considered.

In this study, the method is proposed in which the rule of friction between the grains and wall is taken into consideration besides the rule of vertical movement by gravity.

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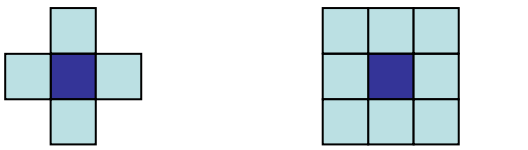
2. PRELIMINARY INVESTIGATION

Several rules of local neighborhood have ever been proposed. Neumann's neighborhood and Moore's neighborhood, which are shown in Fig.1, are generally used in two-dimensional analysis in CA. Each cell has 4 or 8 neighboring cells. The state of each cell is determined by the state of neighboring cells and rules of state transition. Besides a square, a polygon and solid can be used as the shape of a cell. There are researches[4,6] in which some cells are combined in order to express the roughness of a grain.

Let us simulate granular flow using Neumann's and Moore's neighborhood. A cell corresponds to a grain. It has the state of either occupied by a grain or vacant. The rules of Neumann's and Moore's neighborhood are shown in Table 1 and Table 2, respectively. A grain is to move downward by gravity but not upward. If there is a possibility to move to more than two cells, random numbers are used to select one cell.

The results of simulation of hourglass using the both rules are shown in Fig.2 and Fig.3. In the case of using Neumann's neighborhood, a crest is not formed. A grain moves to outside left or right when the cell just below is occupied by another grain. This disturbs to form a crest. On the other hand, a crest is formed in the simulation using Moore's neighborhood. Judging from this preliminary investigation, Moore's neighborhood is better to be used as the basic rule.



(a)Neumann's neighborhood (b) Moore's neighborhood

Fig.1 Neighboring cell considered in CA

3. EXPERIMENTS

1) Experiment of granular flow

The vinyle chloride pipe with no bottom (50mm in height and 71.5mm in diameter), which is shown Photo.1, is filled up with rice grains. When it is pulled up, rice grains scatter and form a crest as shown in Photo.2. The height and width of the crest are measured, which are to be the target values of the simulation. PPC paper with grid scale is used as the floor. Photo.2 shows an example of the experiment. Table 3 shows the results of the measurement of experiments. The dispersion of measurement is small. The average of height and width is 30mm and 155mm, respectively.

Table 1 Rule of Neumann's neighborhood

Move down when the cell just below is vacant.	
Move left or right when the cell just below is occupied and the cells of both left and right are vacant.	
Move left when the cell just below and right cell are occupied, and the left cell is vacant.	
Move right when the cell just below and left cell are occupied, and the right cell is vacant.	

Table 2 Rule of Moore's neighborhood

Move down when the cell just below is vacant.	
Move to lower left or lower right when the cells are vacant except for the cell just below.	
Move to lower right when the cell just blow and left cell are occupied.	
Move to lower left when the cell just blow and right cell are occupied.	
Move to lower left when the cell just blow and lower right cell are occupied.	
Move to lower right when the cell just blow and lower left cell are occupied.	

2) Experiment to estimate coefficient of friction

Consideration of horizontal force and friction plays an important role in this study. Since the coefficient of friction of rice grains in not known, the experiments are carried out to estimate it. For this kind of experiment, the box shear test, which is shown Photo.3, is generally used. But the box shear test was not successful because of the size of rice grain. Then the experiment is adopted which is shown in Photo.4.

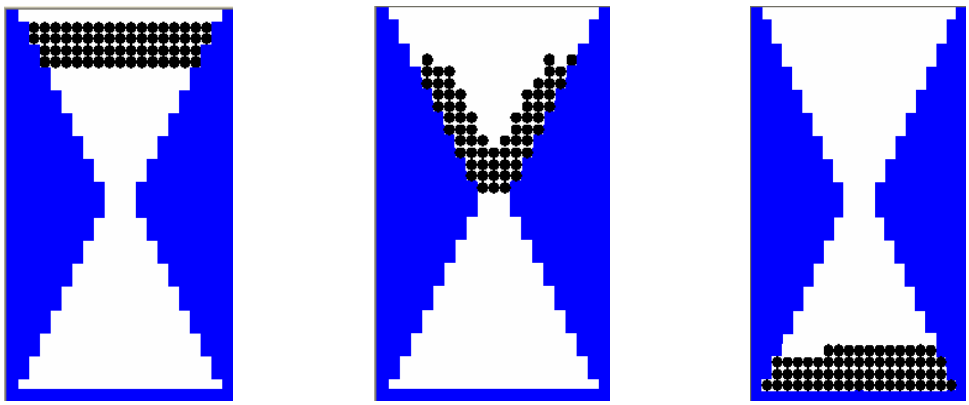


Fig.2 Simulation of hourglass using the rule of Neumann's neighborhood

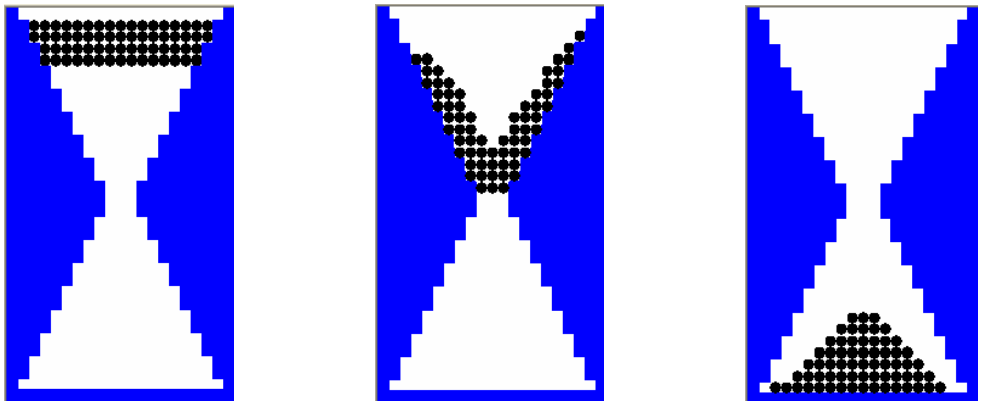


Fig.3 Simulation of hourglass using the rule of Moore's neighborhood

The angle that rice grains start to slide down is used to estimate the coefficient of friction. PPC paper and the paper on which rice grains are stuck with glue are used to estimate the coefficients of friction between a grain and floor and between grains, respectively. Table 4 shows the coefficient of friction of a rice grain which is obtained by the experiments. In order to verify the results, similar experiment is carried out for standard sand and the results are compared with those of box shear test. The coefficients of friction between sand grains are 0.832 and 0.839, respectively. Nearly identical values are obtained. So that, the experiment using slope is considered to be effective.

Photo.1 Rice grains in the vinyle chloride pipe

4. RULES OF HORIZONTAL FORCE AND FRICTION

1) Horizontal force

A grain receives the horizontal force as well as vertical load from other grains located upward. The grain located on the slope of the crest slide down when the horizontal force to it exceeds the friction force. Such the effect is not taken into consideration in the Moore's rule shown in Table 2. Fig.4 shows the all states in Moore's neighborhood. For example, let us consider the case ⑦. Since cells are regularly arranged, the weight of a cell (a grain) is transmitted to the cell just below as the vertical force. No horizontal force is transmitted. But in practice there exists the horizontal force pushing grains outside besides the vertical force. Then, as shown in Fig.5, the upper grain is divided into two half-sized grains, and they give horizontal and vertical forces to the grains below as

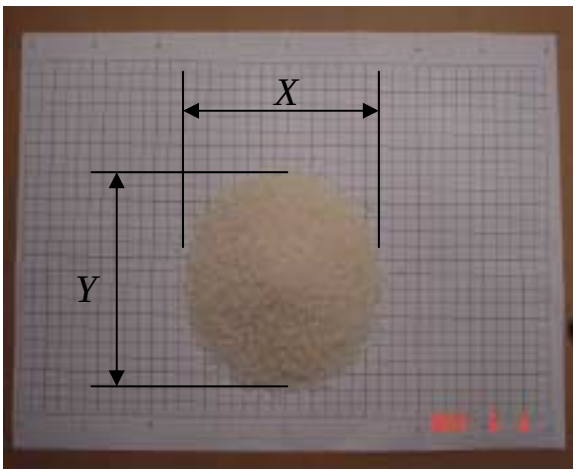


Photo.2 Result of experiment of granular flow

Table 3 Height and width of crest

Cycle	X(mm)	Y(mm)	Z(mm)
1	154	152	30
2	155	156	30
3	155	155	30
4	152	156	29
5	156	156	30
Average	155		30

Table 4 Coefficients of static friction of rice grain

between grains and floor	between grains
0.43	0.9

shown in Fig.6. Applying this idea to the state in Fig.4 respectively, the arrangements shown in Fig.7 are obtained.

2) Rule of friction

The force of friction can be explained using Fig.8. When the horizontal force (H) acts to the grain at rest, it moves if the horizontal force exceeds the maximum static friction force (F). F can be expressed by Eq.(1).

$$F = \mu N \tag{1}$$

where μ and N are the coefficients of static friction and reaction of gravity, respectively. In the simulation, the grain which receives the horizontal force greater than friction force goes off the next vacant cell or lower vacant cell.

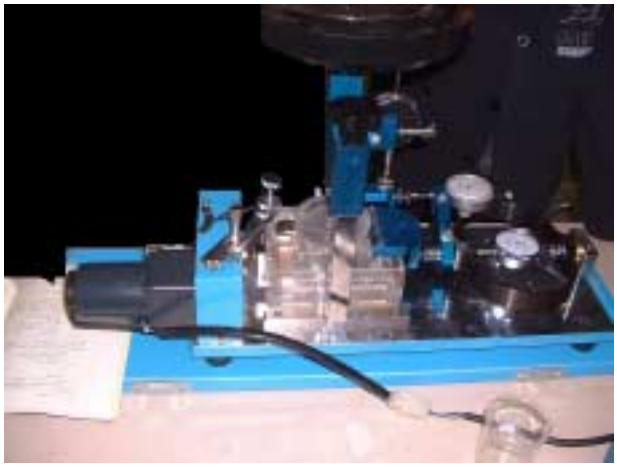


Photo.3 Apparatus of box shear test

Photo.4 Apparatus of experiment to estimate coefficient of rice grains

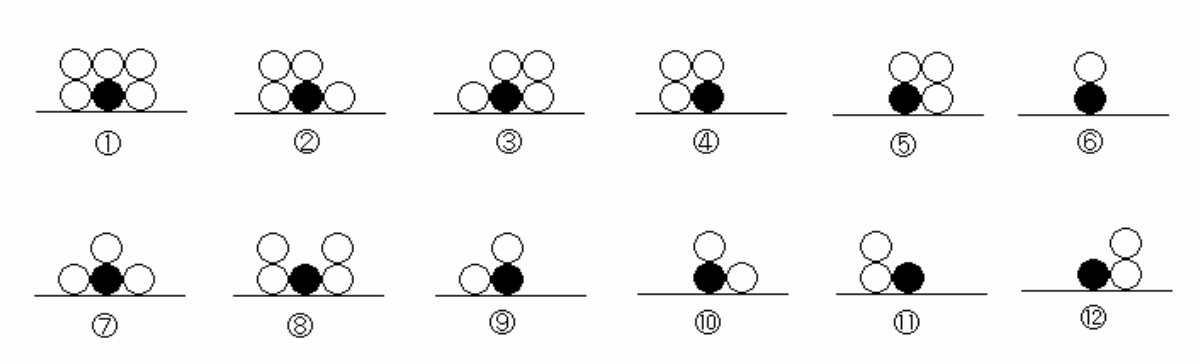


Fig.4 All the states of a grain and its surroundings

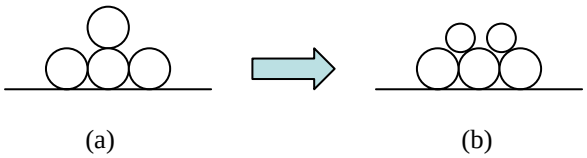


Fig.5 Model for calculating horizontal and vertical force

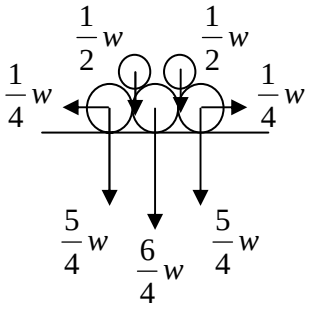


Fig.6 Horizontal and vertical force act to grains

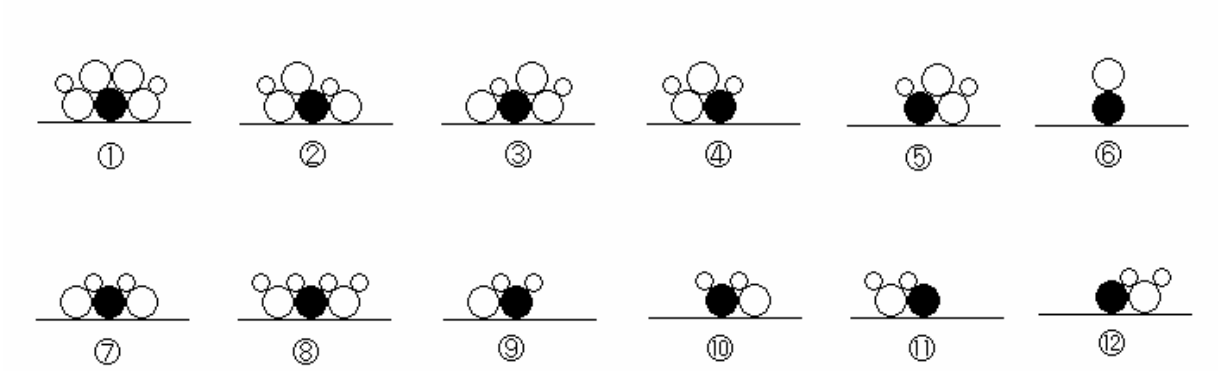


Fig.7 Arrangement of grains for calculation horizontal and vertical force

5. SIMULATIONS

Simulation of the granular flow is carried out. The number of grains in the pipe shown in Photo.1 is estimated as 8,744 from the comparison of the weigh of 100 grains and total weight of grains in the pipe. The capacity of the pipe is about 200,000mm^3. The volume of a grain is 22.9mm^3 (=200,000/8,744). Supposing that the shape of a grain is sphere, the radius of a grain is 1.76mm and the diameter is about 3.5mm from $4/3\ \pi\ r^3$=22.9mm^3. In the following two-dimensional simulation, a grain is expressed as the

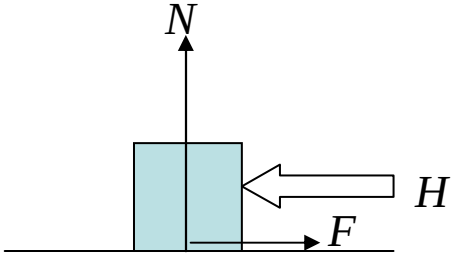


Fig.8 Friction force and horizontal force

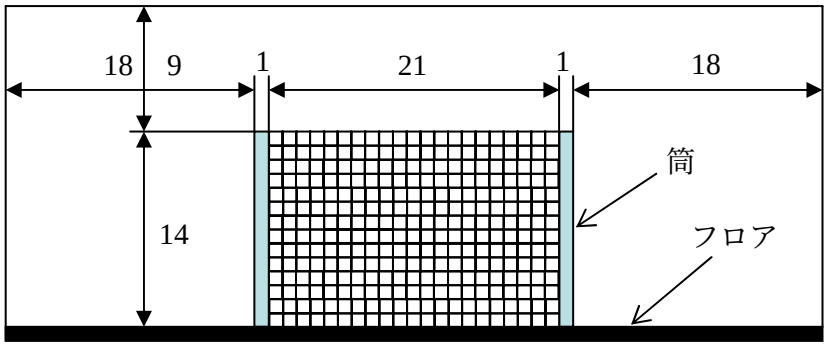


Fig.9 Analytical model

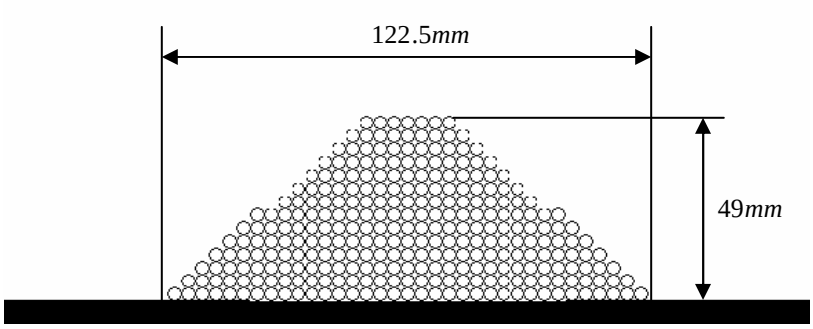


Fig.10 Results of simulation considering no horizontal force and friction

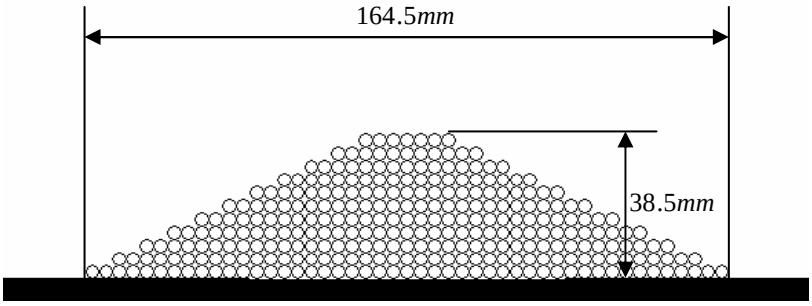


Fig.11 Results of simulation considering horizontal force and friction

square cell of 3.5mm×3.5mm. The vertical section with maximum area has 3,575mm^2 (=71.5mm × 50.0mm). 292 (=3,575mm^2/3.5mm^2) grains are supposed to be contained in it. The vertical section can be modeled by the cells of 21 (=71.5mm/3.5mm) columns × 14 (=50mm/3.5mm) rows. In the analytical model 294 cells are contained which is shown in Fig.9. The figures in Fig.9 indicate the number of cells. Fig.10 shows the result of simulation (Simulation Ⅰ) in which the effect of horizontal force and friction are not considered. Fig.11 shows the result of simulation (Simulation Ⅱ) by the method presented in this study. The height and width of the crest in the simulations are shown in Table 5. In the simulation without consideration of horizontal force and friction, the crest is higher and smaller than those of the experiment. On the other hand, using the method presented in this study, the result of simulation explains the experiment with fairly good approximation. If examined in detail, both the height and width of the crest are a little larger than those of the experiment. It seems to be a limit in ability of two-dimensional analysis.

Table 5 Comparison of the results of experiment and simulations

	width of crest (mm)	height of crest (mm)
Experiment	155.0	30.0
Simulation Ⅰ	122.5	38.5
Simulation Ⅱ	164.5	49.0

The three-dimensional analysis may be necessary intending to develop the accuracy of simulation. Moreover, the introduction of the coefficient of dynamic friction is expected besides the coefficient of static friction.

5. CONCLUSIONS

The new method is proposed to simulate granular flow in which the effect of horizontal force and friction are properly introduced.

The major results are as follows.

1) The coefficient of friction of rice grains used in this study was estimated by the experiments.
2) The new method based on Moore's neighborhood was proposed in which horizontal force to grains and friction are properly considered.
3) Simulation using the method presented in this study could explain the results of experiment of granular flow with fairly good approximation.

The motivation of this research is to apply CA to the analysis of flow of fresh concrete. But it becomes clear that some problems are to be solved. The development of the system which enables the three-dimensional simulation is inevitable in order to improve the accuracy. It is also important subject to incorporate the mechanism of flow of fresh concrete into the rule of CA.

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