

# CHAPTER 6: ULTIMATE LIMIT STATE

## 6.1 GENERAL

It shall be in accordance with JSCE Standard Specification (Design), 6.1. The collapse mechanism in statically indeterminate structures shall not be considered.

### [COMMENT]:

As yielding does not take place in CFRM, the collapse mechanism due to the formation of plastic hinges shall generally not be considered. The effects of steel reinforcement on member capacity when CFRM is used in conjunction with steel reinforcement may be calculated according to JSCE Standard Specification (Design), 6.2 to 6.4.

## 6.2 SAFETY VERIFICATION OF BENDING MOMENT AND AXIAL FORCE

### 6.2.1 Design capacity of member cross-section

(1) In members subjected to axial compressive force, the upper limit of axial compressive capacity  $N'_{oud}$  shall be calculated according to Eq. (6.2.1) when ties are used, and according to Eq. (6.2.1) or Eq. (6.2.2) whichever that gives the larger result when spiral reinforcement is used.

$$N'_{oud} = 0.85f'_{cd}A_c/\gamma_b \quad (6.2.1)$$

$$N'_{oud} = (0.85f'_{cd}A_e + 2.5E_{sp}\epsilon_{fspd}A_{spe})/\gamma_b \quad (6.2.2)$$

where

$A_c$  : cross-sectional area of concrete

$A_e$  : cross-sectional area of concrete enclosed by spiral reinforcement

$A_{spe}$  : equivalent cross-sectional area of spiral reinforcement ( $=\pi d_{sp}A_{sp}/s$ )

$d_{sp}$  : diameter of concrete section enclosed by spiral reinforcement

$A_{sp}$  : cross-sectional area of spiral reinforcement

$s$  : pitch of spiral reinforcement

$f'_{cd}$  : design compressive strength of concrete

$E_{sp}$  : Young's modulus of spiral reinforcement ( $E_{fu}$ )

$\epsilon_{fspd}$  : design value for strain of spiral reinforcement in ultimate limit state, may generally be taken as  $2000 \times 10^{-6}$ . If the design strength  $f_{fbd}$  is less than  $E_{sp}\epsilon_{fspd}$  when the spiral reinforcement is regarded as a bent portion,  $E_{sp}\epsilon_{fspd}$  shall be substituted for  $f_{fbd}$ .

$\gamma_b$  : Member factor, generally taken to be 1.3

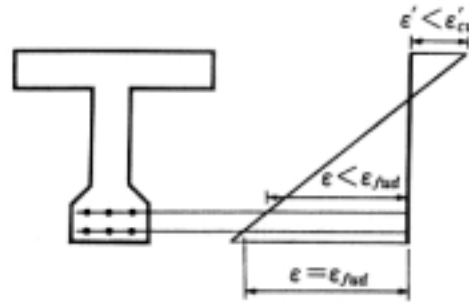
(2) When the bending moment and the design capacity of member cross-sections are calculated according to the direction of section force for unit width of member sections or members, calculations shall be performed on the basis of assumptions (i) to (iii) given below.

(i) Fiber strain is proportional to the distance from the neutral axis.

(ii) Tensile stress of concrete is ignored.

(iii) The tensile force - strain curve of the CFRM follows **3.4.3**.

(3) For fiber rupture flexural failure, the capacity when any reinforcement reaches design ultimate strain  $\epsilon_{fud}$  as shown in **Fig. 6.2.1** is taken to be the design capacity of the member cross-sections. The member factor  $\gamma_b$  may generally be taken as 1.15 to 1.3.



**Fig. 6.2.1 Strain condition at fiber rupture flexural failure in members with multi-layer reinforcement**

(4) For flexural compression failure, the compressive stress distribution in the concrete may be assumed to be identical to the rectangular compressive stress distribution (equivalent stress block) given in JSCE Standard Specification (Design), 6.2.1(3). The member factor  $\gamma_b$  may generally be taken as 1.3.

(5) The design capacity of a member cross-section subjected to combined biaxial bending moment and axial forces shall be calculated according to (2) to (4) explained above.

(6) When the effect of axial forces is negligible, the cross-sectional capacity may be calculated as for a flexural member. Axial forces may be taken to be negligible when  $e/h \geq 10$ , where  $h$  is section height and eccentricity  $e$  is the ratio of design flexural moment  $M_d$  to design axial compressive force  $N'_d$ .

**[COMMENTS]:**

Particularly when high ductility is required, measures such as combining CFRM with steel reinforcement, confinement of compression zone concrete etc., have to be implemented.

(1) As the compressive strength of CFRM is lower than the tensile strength and subject to significant variation, the effects of compressive strength are to be ignored for the purposes of calculation of axial compressive capacity  $N'_{oud}$ .

The effects of using CFRM for spiral reinforcement are allowed for in Eq (6.2.2). The design value  $\epsilon_{fspd}$  for the strain of spiral reinforcement at ultimate limit state has been set at  $2000 \times 10^{-6}$ , allowing for the fact that in the equation for axial compression capacity when steel reinforcement is used, the steel is assumed to yield on the basis of test results. If the design strength when spiral reinforcement is regarded as a bent portion  $f_{fbd}$  is lower than  $E_{sp}\epsilon_{fspd}$ , the latter may be substituted.

(3) As there is no yielding and no plastic region when CFRM is used, rupture begins from reinforcing materials when the strain of the reinforcement reaches the ultimate strain. The first rupturing of the reinforcing material is thus generally simultaneous with the ultimate state of the member, and capacity is calculated from the strain distribution obtained assuming plane sections remain plane. In a member with steel reinforcement arranged in multiple layers, stress may be evaluated from the position of the center of gravity of the steel, but for CFRM, as **Fig. 6.2.1** illustrates, fiber rupture flexural failure takes place when the outermost reinforcement reaches the ultimate strain. If different types of CFRM are

used within the same section, or if bonded and unbonded reinforcing material is used together, these circumstances must be allowed for in calculating the capacity.

(4) In flexural compression failure, it is possible to calculate capacity in the same way as for steel, therefore calculation of capacity using the equivalent stress block method is allowed here.

## 6.2.2 Structural detail

### (1) Minimum axial reinforcement

(i) In concrete members reinforced with CFRM where axial forces are dominant, the quantity of axial reinforcement shall be not less than  $0.8(E_0/E_{fu})\%$  of the calculated minimum cross-sectional area of the concrete, where  $E_0$  is reference Young's modulus ( $=200 \text{ kN/mm}^2$ ), and  $E_{fu}$  is Young's modulus of axial reinforcement. The "calculated minimum cross-sectional area of the concrete" here refers to the minimum cross-sectional area of concrete required for axial support only.

Where the section is larger than the minimum required section, the amount of axial reinforcement should preferably be in excess of  $0.1(E_0/E_{fu})\%$  of the concrete cross-sectional area.

(ii) The ratio of tensile reinforcement in beam members where the effects of bending moment are dominant shall generally be not less than  $(35 f_{tk}/f_{fuk})\%$  or 0.2%, whichever is the greater. For T-cross sections, the amount of axial tensile reinforcement shall be not less than 1.5 times as great as the above value, relative to the effective cross-sectional area of the concrete. In this,  $f_{tk}$  is the characteristic value of the tensile strength of the concrete, and  $f_{fuk}$  is the characteristic value of the tensile strength of the tensile reinforcement. The "effective cross-sectional area of the concrete" here refers to the effective depth of the section  $d$  multiplied by the web width  $b_w$ .

### (2) Maximum axial reinforcement

In concrete members where axial forces are dominant, the amount of axial reinforcement shall generally be not greater than  $6(E_0/E_{fu})\%$  of the cross-sectional area of the concrete.

## [COMMENTS]:

### (1)

(i) The compressive strength of CFRM can be ignored for the purpose of calculating axial compressive capacity, but in order to ensure axial rigidity, a minimum amount of axial reinforcement has been specified, as for steel reinforcement. Where the member cross section is larger than the calculated minimum cross-sectional area of the concrete, while a minimum axial reinforcement is required from the point of view of cracking, as CFRM is not liable to corrosion, the requirements given here have been relaxed slightly as compared to those for steel reinforcement. Where CFRM is used in conjunction with steel, however, the value of (steel quantity +  $(E_{fu}/E_0) \cdot$  CFRM quantity) must be not less than 0.15% of the cross-sectional area of the concrete.

(ii) Where the ratio of tensile reinforcement is extremely low, the reinforcement ruptures as soon as cracking appears, inducing a state of brittle failure. The minimum amount of reinforcement is prescribed in order to avoid this. Allowing for the size effect of the member, the minimum tensile reinforcement ratio may be either  $(35 k_1 f_{tk}/f_{fuk})\%$  or 0.2%, whichever is the greater.  $k_1$  is obtained from Eq. (C 6.2.1).

$$k_1 = 0.6 / (h^{1/3}) \quad (\text{C 6.2.1})$$

where  $h$  is total member depth (m), provided that  $0.4 \leq k_1 \leq 1.0$ .

## 6.3 SAFETY VERIFICATION OF SHEAR FORCES

### 6.3.1 General

It shall be in accordance with JSCE Standard Specifications (Design), 6.3.1.

### 6.3.2 Design shear forces of beam members

It shall be in accordance with JSCE Standard Specifications (Design), 6.3.2.

### 6.3.3 Design shear capacity of beam members

(1) Design shear capacity  $V_{ud}$  is obtained from Eq. (6.3.1), provided that when bent-up reinforcement and stirrups are used together for shear reinforcement, the stirrups bear not less than 50% of shear force required to be borne by the shear reinforcement.

$$V_{ud} = V_{cd} + V_{sd} + V_{ped} \quad (6.3.1)$$

where

$V_{cd}$  : design shear capacity of beam members not used in shear reinforcement, obtained from Eq. (6.3.2).

$$V_{cd} = \beta_d \cdot \beta_p \cdot \beta_n \cdot f_{vcd} \cdot b_w \cdot d / \gamma_b \quad (6.3.2)$$

where

$$f_{vcd} = 0.2\sqrt[3]{f'_{cd}} \quad (\text{N/mm}^2), \text{ provided that } f_{vcd} \leq 0.72 \text{ N/mm}^2 \quad (6.3.3)$$

$$\beta_d = \sqrt[4]{1/d} \quad (d:\text{m}); \text{ if } \beta_d > 1.5 \text{ then } \beta_d = 1.5$$

$$\beta_p = \sqrt[3]{100p_w E_{fu} / E_0}; \text{ if } \beta_p > 1.5 \text{ then } \beta_p = 1.5$$

$$\beta_n = 1 + M_0/M_d; \text{ (if } N'_d \geq 0); \text{ if } \beta_n > 2 \text{ then } \beta_n = 2$$

$$1 + 2 M_0/M_d \text{ (if } N'_d < 0); \text{ if } \beta_n < 0 \text{ then } \beta_n = 0$$

$N'_d$  : design axial compressive force

$M_d$  : design bending moment

$M_0$  : bending moment required to cancel out stresses set up by axial forces in the tensioned edge, relative to design bending moment  $M_d$

$E_{fu}$  : Young's modulus of tensile reinforcement

$E_0$  : reference Young's modulus (=200 kN/mm<sup>2</sup>)

$b_w$  : width of web

$d$  : effective depth

$$p_w = A_f / (b_w d)$$

$A_f$  : cross-sectional area of tensile reinforcement

$f'_{cd}$  : design compressive strength of concrete, in units of N/mm<sup>2</sup>

$\gamma_b$  : generally = 1.3

$V_{sd}$  : design shear capacity borne by shear reinforcement, obtained from Eq. (6.3.4)

$$V_{sd} = \left[ A_w E_w \varepsilon_{fwd} (\sin \alpha_s + \cos \alpha_s) / s_s + A_{pw} \sigma_{pw} (\sin \alpha_p + \cos \alpha_p) / s_p \right] z / \gamma_b \quad (6.3.4)$$

$A_w$  : total cross-sectional area of shear reinforcement in section  $s_s$

$E_w$  : Young's modulus of shear reinforcement (=  $E_{fu}$ )

$\varepsilon_{fwd}$  : design value of shear reinforcement strain in ultimate limit state, obtained from Eq. (6.3.5). Where  $E_w \varepsilon_{fwd}$  is greater than the design value for the strength of the bent portion  $f_{fbd}$ ,  $f_{fbd}$  is substituted for  $E_w \varepsilon_{fwd}$ .  $f_{fbd}$  may be obtained from Eq. (3.4.1).

$$\varepsilon_{fwd} = \sqrt{f'_{mcd} \frac{P_w E_{fu}}{P_{web} E_w} \left[ 1 + 2 \left( \frac{\sigma'_N}{f'_{mcd}} \right) \right]} \times 10^{-4} \quad (6.3.5)$$

$\alpha_s$  : angle formed by shear reinforcement and member axis

$s_s$  : spacing of shear reinforcement

$P_{web}$  :  $A_w / (b_w \cdot s_s)$

$A_{pw}$  : total cross-sectional area of shear reinforcement tendons in section  $s_p$

$\sigma_{pw}$  : effective tensile stress of shear reinforcement

$$\sigma_{pw} = \sigma_{wpe} + E_{fpw} \varepsilon_{fwd} \leq f_{fpud}$$

$\sigma_{wpe}$  : effective tensile stress of shear reinforcement tendons

$E_{fpw}$  : Young's modulus of shear reinforcement

$f_{fpud}$  : design tensile strength of shear reinforcement

$\alpha_p$  : angle formed by shear reinforcement and member axis

$s_p$  : spacing of shear reinforcement

$z$  : distance from point of action of compressive stress resultant force, generally  $d / 1.15$

$\sigma'_N$  : average axial compressive stress

$$\sigma'_N = (N'_d + P_{ed}) / A_g$$

$$\text{if } \sigma'_N > 0.4f'_{mcd} \text{ then } \sigma'_N = 0.4f'_{mcd}$$

$P_{ed}$  : effective tensile force in axial tendons

$A_g$  : total cross-sectional area

$f'_{mcd}$  : design compressive strength of concrete allowing for size effect (N/mm<sup>2</sup>)

$$f'_{mcd} = \left( \frac{h}{0.3} \right)^{-1/10} \cdot f'_{cd}$$

$f'_{cd}$  : design compressive strength of concrete, in N/mm<sup>2</sup>

$h$  : member depth (m)

$\gamma_b$  : generally = 1.15

$V_{ped}$  : component of effective tensile force of axial tendons parallel to shear force, obtained from Eq. (6.3.6)

$$V_{ped} = P_{ed} \sin \alpha_p / \gamma_b \quad (6.3.6)$$

$\alpha_p$  : angle formed by shear reinforcement and member axis

$\gamma_b$  : generally = 1.15

(2) When beam members are supported directly,  $V_{ud}$  need not be investigated for the zone from the support face to one-half of the depth  $h$  of the members. In this zone, shear reinforcement more than the

minimum required shall be placed in the cross section from the support face to  $h/2$ . In members of non-uniform section, the depth at the support face may be adopted as the member depth; parts of a haunch where the gradient is less than 1:3 shall be considered to be effective.

(3) The design diagonal compressive capacity  $V_{wcd}$  of web concrete to shear force shall be obtained from Eq. (6.3.7).

$$V_{wcd} = f_{wcd} \cdot b_w \cdot d / \gamma_b \quad (6.3.7)$$

where

$$f_{wcd} = 1.25\sqrt{f'_{cd}} \text{ (N/mm}^2\text{); provided that } f_{wcd} \leq 7.8 \text{ N/mm}^2$$

$$\gamma_b = \text{generally} = 1.3$$

(4) Web width of members

(i) Where the diameter of a single duct in prestressed concrete members is equal to or greater than 1/8 of the web width, the web width assumed in Eq. (6.3.2) shall be smaller than the actual web width  $b_w$ . In such a case, the web width may generally be reduced by the total of the diameters of the ducts  $\phi$  arranged in that section, giving  $b_w - 1/2\Sigma\phi$ .

(ii) For members with web widths varying in the direction of member depth, other than those with circular sections, the minimum width  $b_w$  within the range of effective depth  $d$  shall be adopted. For members with multiple webs,  $b_w$  shall be the total width of all webs. For solid or hollow circular sections, web width  $b_w$  shall be either the length of one side of a square with the equivalent area, or as the total width of webs of square boxes having the same area. In these cases, the area of axial tensile reinforcement  $A_f$  shall be the cross-sectional area of reinforcement in 1/4 (90°) of the cross section of the tensioned side. The effective depth  $d$  shall be either the distance from the compression edge of the square or box of equivalent area to the centroid of the reinforcement, accounted for as  $A_f$ . These definitions of axial tensile reinforcement area shall not apply in calculation of flexural capacity.

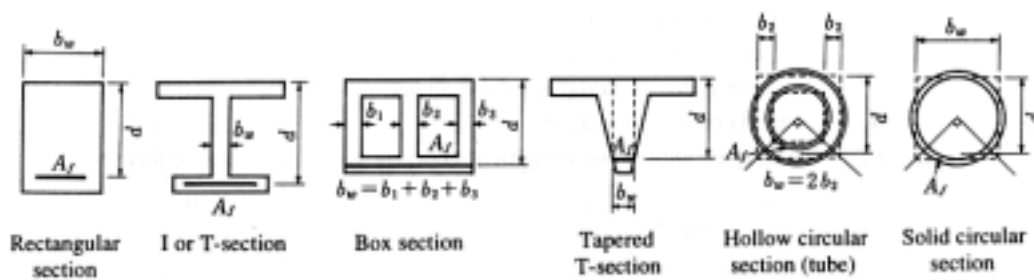


Fig. 6.3.1 Definitions of  $b_w$  and  $d$  for various cross-sections

**[COMMENTS]:**

(1) The design shear force  $V_{ud}$ , as shown in Eq. (6.3.1), is given as the sum of the components carried by the concrete  $V_{cd}$  and by the shear reinforcement  $V_{sd}$ , except that the components ( $V_{ped}$ ) of effective tensile force in the axial reinforcement parallel to the shear force is ignored.

Previous studies indicate that the shear capacity of beam members with CFRM used for tensile reinforcement but without shear reinforcement can generally be evaluated by taking into account the axial rigidity of the tensile reinforcement.  $V_{cd}$  is thus calculated according to the equation used for steel, allowing for the ratio of the Young's modulus of CFRM to that of steel.

The strain  $\varepsilon_{fwd}$  of shear reinforcement at the ultimate limit state is affected by concrete strength, the rigidity of tensile and shear reinforcement, and axial compression force. These functions are given by Eq. (6.3.5). Eq. (6.3.5) is derived from the most recent findings of research on the design shear capacity of beam members using CFRM, shown below. These findings offer a more accurate method than the conventional one for estimating shear stress, by incorporating a more realistic shear resistance mechanism. This method may be followed in estimating the ultimate shear capacity.

The shear capacity obtained by the method given below is generally greater than that obtained from Eq. (6.3.1). The method below is greatly simplified, for instance by conservatively ignoring the effect of the shear span-to-depth ratio on shear capacity, but in some instances it will give a lower shear capacity than Eq. (6.3.1), for example when the main reinforcement has high rigidity.

Design shear capacity when shear reinforcement does not break is calculated as follows:

$$V_{ud} = V_{cd} + V_{sd} \quad (\text{C 6.3.1})$$

where

$V_{cd}$  = design shear force carried by concrete, obtained from Eq. (C 6.3.2)

$$V_{cd} = V_{czd} + V_{aid} \quad (\text{C 6.3.2})$$

where

$V_{czd}$  : design shear force carried by concrete in compression zone, obtained from Eq.

(C 6.3.3)

$$V_{czd} = \beta f'_{mcd} x_e b_w / \gamma_b \quad (\text{C 6.3.3})$$

$V_{aid}$  : design shear force carried by concrete in diagonal cracking zone, obtained from Eq.

(C 6.3.4)

$$V_{aid} = \beta_P \beta_{pE} f'_{mcd}{}^{1/3} (h - x_e) b_w / \gamma_b \quad (\text{C 6.3.4})$$

$V_{sd}$  = shear capacity carried by shear reinforcement, obtained from Eq. (C 6.3.5)

$$V_{sd} = A_w E_w \varepsilon_{fwd} (h - x_e) b_w / (\tan \theta_{cr} s_s) / \gamma_b \quad (\text{C 6.3.5})$$

$x_e$  : depth of concrete compression zone at ultimate, obtained from Eq. (C 6.3.6)

$$x_e = [1 - 0.8(p_{web} E_{fw})^{-0.2} \left[ 1 + \left( \frac{\sigma'_N}{f'_{mcd}} \right)^{0.7} \right]] x \quad (\text{C 6.3.6})$$

$\varepsilon_{fwd}$  : strain in shear reinforcement at ultimate limit state, obtained from Eq. (C 6.3.7)

$$\varepsilon_{fwd} = 0.0001 \sqrt{f'_{mcd} \frac{p_w E_{fu}}{p_{web} E_w} \left[ 1 + 2 \left( \frac{\sigma'_N}{f'_{mcd}} \right) \right]} \quad (\text{C 6.3.7})$$

$\theta_{cr}$  : angle of diagonal cracking, obtained from Eq. (C 6.3.8)

$$\theta_{cr} = 45 \left[ 1 - \left( \frac{\sigma'_N}{f'_{mcd}} \right)^{0.7} \right] \quad (\text{C 6.3.8})$$

$$\beta = 0.2 \left( \frac{\sigma'_N}{f'_{mcd}} \right)^{0.7}$$

$$\beta_P = 1 - 5 \frac{\sigma'_N}{f'_{mcd}}; \text{ if } \beta_P < 0 \text{ then } \beta_P = 0$$

$$\beta_{pE} = 0.24 \left( \frac{p_w E_{fu} + 10 p_{web} E_w}{5000k} + 0.66 \right); \text{ if } \beta_{pE} > 0.40 \text{ then } \beta_{pE} = 0.40$$

$$k = 1 - \left( \frac{\sigma'_N}{f'_{mcd}} \right)^{0.1}$$

$f'_{mcd}$  : design compressive strength of concrete, allowing for size effect (N/mm<sup>2</sup>)

$$f'_{mcd} = \left( \frac{h}{0.3} \right)^{-1/10} \cdot f'_{cd}$$

$f'_{cd}$  : design compressive strength of concrete (N/mm<sup>2</sup>)

$b_w$  : web width

$d$  : effective depth

$h$  : beam height (m)

$A_f$  : cross-sectional area of tension reinforcement (mm<sup>2</sup>)

$A_w$  : total cross-sectional area of shear reinforcement in zone  $s_s$

$$p_w = A_f / (b_w d)$$

$$p_{web} = A_w / (b_w s_s)$$

$E_{fu}$  : Young's modulus of tension reinforcement (N/mm<sup>2</sup>)

$E_w$  : Young's modulus of shear reinforcement (N/mm<sup>2</sup>)

$\sigma'_N = (N'_d + P_{ed}) / A_g$  (N/mm<sup>2</sup>); if  $\sigma'_N > 0.4 f'_{mcd}$  then  $\sigma'_N = 0.4 f'_{mcd}$

$N'_d$  : design axial compression force

$P_{ed}$  : effective tensile force of axial reinforcement

$A_g$  : cross-sectional area of entire section

$s_s$  : spacing of shear reinforcement

$x$  : position of neutral axis according to elastic theory, ignoring tension section

$\gamma_b$  : generally = 1.3

Design shear capacity when shear reinforcement breaks by fiber rupture is calculated as follows:

$$V_{ud} = V_{c0} - \beta_m (V_{c0} - V_{czd}) + \beta_m V_{aid} + \beta_m V_{sd} \quad (\text{C 6.3.9})$$

where

$V_{c0}$  : load at which diagonal cracking occurs, obtained from Eq. (C 6.3.10)

$$V_{c0} = \beta_0 \beta_d^2 \beta_{cd} x_0 b_w / \gamma_b + \beta_{P0} \beta_{PE0} \beta_d f'_{cd}{}^{1/3} (h - x_0) b_w / \gamma_b \quad (\text{C 6.3.10})$$

$V_{czd}$  : design shear force carried by concrete in compression zone; may be obtained from Eq. (C 6.3.3)

$V_{aid}$  : design shear force carried by concrete in diagonal cracking zone; may be obtained from Eq. (C 6.3.4)

$V_{sd}$  : design shear force carried by shear reinforcement; may be obtained from Eq. (C 6.3.5)

$x_0$  : depth of compression zone in concrete at onset of diagonal cracking, obtained from Eq. (C 6.3.11)

$$x_0 = \left[ 1 + \left( \frac{\sigma'_N}{f'_{cd}} \right)^{0.7} \right] x \quad (\text{C 6.3.11})$$

$$\beta_0 = 0.14 \left( \frac{\sigma'_N}{f'_{cd}} \right)^{0.7}$$

$$\beta_d = \sqrt[4]{1000/d} ; \text{ if } \beta_d > 1.5 \text{ then } \beta_d = 1.5$$

$$\beta_{P0} = 1 - 5 \frac{\sigma'_N}{f'_{cd}} ; \text{ if } \beta_{P0} < 0 \text{ then } \beta_{P0} = 0$$



$$\beta_{pE0} = 0.17 \left( \frac{p_w E_{fu}}{5000k} + 0.66 \right); \text{ if } \beta_{pE0} > 0.28 \text{ then } \beta_{pE0} = 0.28$$

$$k = 1 - \left( \frac{\sigma'_N}{f'_{cd}} \right)^{0.7}$$

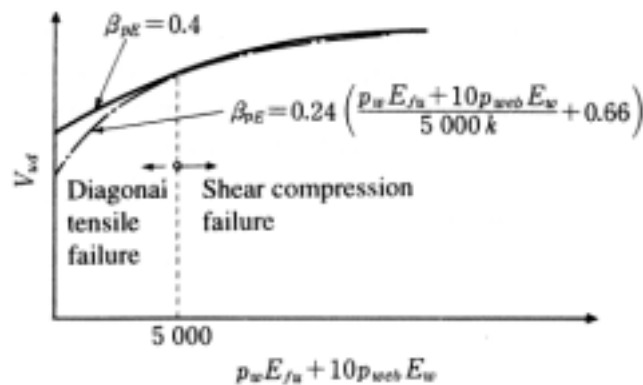
$$\beta_m = \frac{f_{ud}}{E_w \varepsilon_{fud}}$$

$f_{ud}$  : design tensile strength of shear reinforcement, taken as equivalent to design strength of bent portion  $f_{fbd}$ , where  $f_{fbd}$  may be obtained from Eq. (3.4.1)

Design shear capacity  $V_{ud}$ , as shown in Eq. (C 6.3.1), is expressed as the sum of the components carried by concrete  $V_{cd}$  and by shear reinforcement  $V_{sd}$ . For each of these components, the effects of the Young's modulus of the tendons are evaluated as the rigidity obtained by multiplying the reinforcement ratio by the Young's modulus of the reinforcing material.

The shear force carried by the concrete in the compression zone increases as the axial compression force increases. This is expressed by Eq. (C 6.3.3).

The mode of failure of the beam varies depending on the rigidity (reinforcement ratio  $\times$  Young's modulus) of the main reinforcement and the shear reinforcement. That is, as the rigidity of the main reinforcement and the shear reinforcement increases, the failure mode shifts from diagonal tensile failure to shear compressive failure.  $\beta_{pE}$  in Eq. (C 6.3.4) signifies that when the rigidity of the main reinforcement and the shear reinforcement is low and the beam undergoes diagonal tensile failure, the shear transmission stress of the diagonal cracking zone increases as the rigidity of the main reinforcement and the shear reinforcement increases. However, when the rigidity is high and the beam undergoes shear compressive failure, the shear transmission force of the concrete in the diagonal cracking zone remains constant regardless of the rigidity of the main reinforcement and the shear reinforcement (**Fig. C 6.3.2**).



**Fig. C 6.3.2 Effect of rigidity of longitudinal and shear reinforcement on shear strength**

The shear span-to-depth ratio also affects the mode of failure, although previous studies have confirmed that at shear span-to-depth ratios of 2 or more, if the reinforcement has low rigidity, diagonal tensile failure will occur. Where axial compressive force is present, the mode of failure shifts from diagonal tensile failure to shear compressive failure. Previous studies have confirmed that shear compressive failure occurs even at low reinforcement rigidity, and the term  $k$  in  $\beta_{pE}$  (Eq. C 6.3.4) is

included to allow for this effect. That is, the reference value ( $p_w E_{fu} + 10p_{web} E_w = 5000$ ) for the case where axial compressive force is not acting, decreases as the axial compressive force increases.

The angle of diagonal cracking, i.e. the angle of the truss diagonals, becomes shallower as the axial compressive force increases. This is expressed in Eq. (C 6.3.8).

Shear reinforcement is thought to fail if the stress in shear reinforcement at ultimate limit state  $E_w \epsilon_{fwd}$  is greater than the strength of the bent portion  $f_{fb}$ , obtained from Eq. (3.4.1). In this case, the design shear capacity  $V_{ud}$  is obtained from Eq. (C 6.3.9). That is, stress in the shear reinforcement after the onset of diagonal cracking, and components  $V_{czd}$  and  $V_{aid}$ , are thought to vary linearly according to the acting shear force, and components  $V_{czd}$ ,  $V_{aid}$  and  $V_{sd}$  are reduced by a factor  $\beta_m$ , obtained by dividing the failure strength of the shear reinforcement by the shear reinforcement stress  $E_w \epsilon_{fud}$ , calculated assuming non-failure of the shear reinforcement (Fig. C 6.3.3).

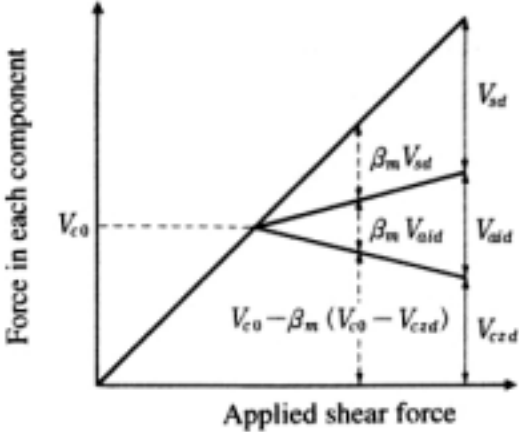


Fig. C 6.3.3 Modeling of each component of shear capacity

The method given here for calculation of shear capacity is derived from dynamic models agreeing with empirical facts, such as that the angle of the main compressive stress within the concrete is not  $45^\circ$  even if the angle of shear cracking within the shear span is generally  $45^\circ$  relative to the member axis, and that the load stress of the concrete carried outside of the truss mechanism varies with the acting shear force, and its value is not equivalent to the shear capacity of members without shear reinforcement. Eq. (C 6.3.5) which follows this method gives the shear force carried by shear reinforcement straddling diagonal cracks; where axial forces are not present, the angle of diagonal cracking is  $45^\circ$ , and the expression approximates the equation given in the JSCE Standard Specification, and also Eq. (6.3.4) of the present Recommendation. The difference between the two equations is that Eq. (C 6.3.5) incorporates a term  $(h-x_e)$  expressing the depth of the diagonal cracking zone, whereas Eq. (6.3.4) incorporates a term  $z$  expressing the arm length of the truss. According to the model referred to above, shear forces other than those carried by the truss mechanism are expressed by  $V_{czd}$  in Eq. (C 6.3.3), and this value generally varies with the acting shear force (cf. Fig. C 6.3.3). The sum of this term  $V_{czd}$  and  $V_{aid}$ , the shear force transmitted by the interlocking of the aggregate in the diagonal cracking zone etc. (cf. Eq. (C 6.3.4)), is generally constant, corresponding closely with Eq. (6.3.2).

(3) The width of diagonal cracking is thought to be wider when CFRM is used than when steel reinforcement is used. The compressive capacity and rigidity of concrete where cracking is present

decreases as the strain perpendicular to the cracks increases, therefore diagonal compressive failure capacity is thought to be lower than when steel reinforcement is used. This hypothesis is yet to be confirmed experimentally, however, and in the present specifications, diagonal compressive capacity of reinforced concrete beams is evaluated conservatively in Eq. (6.3.7).

### 6.3.4 Design punching shear capacity of planar members

(1) When the loaded area is positioned far from free edges or openings, and the eccentricity of the load is small, the design punching shear capacity  $V_{pcd}$  may be determined by Eq. (6.3.8).

$$V_{pcd} = \beta_d \cdot \beta_p \cdot \beta_r \cdot f_{pcd} \cdot u_p d / \gamma_b \quad (6.3.8)$$

where

$$f_{pcd} = 0.2 \sqrt{f'_{cd}} \text{ (N/mm}^2\text{)}; f_{pcd} \text{ shall be } \leq 1.2 \text{ N/mm}^2 \quad (6.3.9)$$

$$\beta_d = \sqrt[4]{1/d} \text{ (d:m)}; \text{ if } \beta_d > 1.5 \text{ then } \beta_d = 1.5$$

$$\beta_p = \sqrt[3]{100 p E_{fu} / E_0}; \text{ if } \beta_p > 1.5 \text{ then } \beta_p = 1.5$$

$$\beta_r = 1 + 1/(1+0.25 u/d)$$

$f'_{cd}$  : design compressive strength of concrete (N/mm<sup>2</sup>)

$u$  : peripheral length of loaded area

$E_{fu}$  : Young's modulus of tensile reinforcement

$E_0$  : standard Young's modulus (=200 kN/mm<sup>2</sup>)

$u_p$  : peripheral length of the design cross-section at  $d/2$  from the loaded area

$d, p$  : effective depth and reinforcement ratio, defined as the average values for the reinforcement in both directions.

$\gamma_b$  : generally = 1.3

(2) When the loaded area is located in the vicinity of free edges or openings in members, the reduction of the punching shear capacity shall be allowed for.

(3) When loads are applied eccentrically to the loaded area, the effects of flexure and torsion shall be allowed for.

#### [COMMENT]:

(1) As with the shear capacity of beam members without shear reinforcement, the punching shear capacity may generally be evaluated by allowing for the axial rigidity of the reinforcement. The Young's modulus of the CFRM is therefore allowed for in the calculation of design punching shear capacity  $V_{pcd}$ .

### 6.3.5 Structural details

(1) In beam members, stirrups not less than  $0.15(E_0/E_{fu})\%$  shall; be arranged over the entire member length, where  $E_0$  is standard Young's modulus (=200 kN/mm<sup>2</sup>), and  $E_{fu}$  is Young's modulus of axial reinforcement. The spacing of the stirrups shall generally be not more than 1/2 of the effective depth of the member, and not more than 30 cm. This provision (1) need not be applied to planar members.

(2) Shear reinforcement equivalent to that required by calculation shall also be arranged in sections equivalent to the effective depth outside of the section where it is required.

(3) The ends of stirrups and bent bars shall be adequately embedded in the concrete on the compressive side.

**[COMMENT]:**

(1) When steel reinforcement is used, stirrups equivalent to not less than 0.15% of the concrete area are installed to prevent sudden failure due to the onset of diagonal cracking. Based on this provision, a minimum amount of stirrup of  $0.15(E_0/E_{fu})\%$  is also imposed here for CFRM reinforcement. As most CFRM have low elasticity and small cross-sectional areas, the spacing requirements given here are slightly stricter than those for steel.

## **6.4 TORSION SAFETY**

### **6.4.1 General**

(1) For structural members not significantly influenced by torsional moment, and those subjected to compatibility torsional moment, the torsional safety studies given in section 6.4 may be omitted. "Structural members not significantly influenced by torsional moment" here refers to members in which the ratio of the design torsional moment  $M_{td}$  to the design pure torsional capacity  $M_{tcd}$ , calculated according to 6.4.2 (members without torsional reinforcement), multiplied by structural factor  $\gamma_s$ , is less than 0.2 for all sections.

(2) When the effects of design torsional reinforcement are not negligible, torsion reinforcement shall be arranged in accordance with 6.4.2.

### **6.4.2 Design torsional capacity**

(1) Torsional capacity in members without torsional reinforcement shall be in accordance with "JSCE Standard Specification (Design)", section 6.4.2.

(2) Torsional capacity in members with torsional reinforcement shall be calculated according to appropriate methods.

**[COMMENT]:**

(2) Studies of CFRM used for torsional reinforcement have not yet been adequately carried out. Design torsional capacity in members with torsional reinforcement must therefore be investigated experimentally and analytically based on reliable techniques.