Method for Calculating the Design Shear Capacity of Reinforced Concrete Beams to Ensure Continuity to the Shear-Span-to-Effective-Depth Ratio

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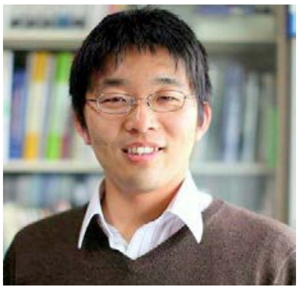

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The safety of reinforced concrete (RC) structures subjected to shear force is verified by confirming that shear forces do not reach the design shear capacities in the standard specifications established by the Japan Society of Civil Engineers [1]. The design shear capacities are calculated using the following equations.

$$V_{yd} = V_{cd} + V_{sd} \tag{1a}$$

$$V_{cd} = \beta_d \cdot \beta_p \cdot f_{vcd} \cdot b_w \cdot d / \gamma_{bc} \tag{1b}$$

$$V_{sd} = \{ A_w \cdot f_{wyd} \cdot (\sin\alpha_s + \cos\alpha_s) / s_s \} \cdot z / \gamma_{bs} \tag{1c}$$

$$f_{vcd} = 0.20\sqrt[3]{f'_{cd}} \leqq 0.72 \ (\mathrm{N/mm^2}) \tag{1d}$$

$$\beta_d = \sqrt[4]{1000/d} \leqq 1.5 \tag{1e}$$

$$\beta_p = \sqrt[3]{100 \cdot p_v} \leqq 1.5 \tag{1f}$$

$$p_v = A_s / (b_w \cdot d) \tag{1g}$$

$$V_{dd} = (\beta_d + \beta_w)\beta_p \cdot \beta_a \cdot f_{dd} \cdot b_w \cdot d / \gamma_{bd} \tag{2a}$$

$$f_{dd} = 0.19\sqrt{f'_{cd}} \tag{2b}$$

$$\beta_p = (1 + \sqrt{100 p_v})/2 \leqq 1.5 \tag{2c}$$

$$\beta_a = 5 / \{1 + (a_v/d)^2\} \tag{2d}$$

$$\beta_w = 4.2\sqrt[3]{100 p_w}\,(a_v/d - 0.75) / \sqrt{f'_{cd}} \geqq 0 \tag{2e}$$

$$p_w = A_w / (b_w \cdot s_s) \tag{2f}$$

When shear reinforcement ratio p_w<0.002, p_w is taken as 0.

The basic experimental equations behind the design equations for calculating the shear capacities are shown below.

$$V_y = V_c + V_s \tag{3a}$$

$$V_c = 0.20 \cdot (p_v \cdot f'_c)^{1/3} \cdot (d/1000)^{-1/4} \cdot \{0.75 + 1.4 / (a/d)\} \cdot b_w \cdot d \tag{3b}$$

$$V_s = \{ A_w \cdot f_{wy} \cdot (\sin\alpha_s + \cos\alpha_s) / s_s \} \cdot z \tag{3c}$$

$$V_d = \frac{0.24 \cdot k \cdot f'^{2/3}_c \cdot (1 + \sqrt{100 p_v}) \cdot (1 + 3.33 r/d)}{1 + (a/d)^2} b_w \cdot d \tag{4a}$$

$$k = 1 + 7.4\sqrt[3]{100 p_w} \cdot (a/d - 0.75) / f'^{2/3}_c \tag{4b}$$

When shear reinforcement ratio p_w<0.002, p_w is taken as 0.

The notations are explained in the standard specifications [1].

The experimental and design equations express the average values and the lower limit of the experimental results, respectively. Figures 1 and 2 show comparisons of V_{yd} and V_{dd} with experimental results V_{uexp} for rectangular cross-sectioned reinforced concrete beams with simple supports. Although V_{yd} and V_{dd} correspond to diagonal tension failure and shear compression failure, respectively, V_{yd} and V_{dd} express the lower limits of V_{uexp}. However, the experimental scopes of p_w are 0~0.72% for diagonal tension failure and 0~2.58% for shear compression failure.

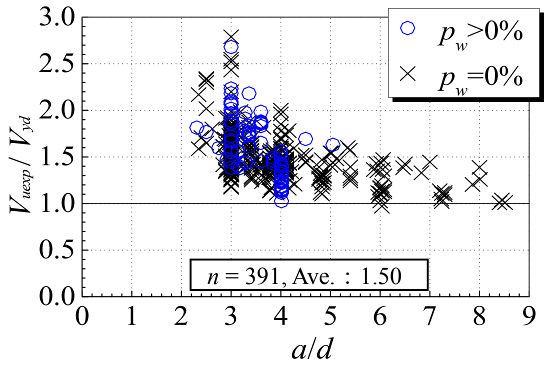

Figure 1. Comparison of V_{yd} and V_{uexp}

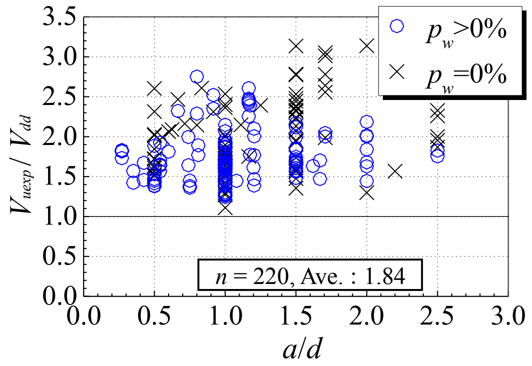

Figure 2. Comparison of V_{dd} and V_{uexp}

Figure 3 shows the relationships of the shear capacities and a/d for a specified section (such as the transverse beams of railway viaducts). Though the equations are expressed as a function of the shear span to the effective depth ratio (a/d), V_{yd} is used in $a/d \geqq 2.0$ and V_{dd} is used in $a/d<2.0$ [1]. V_{yd} and V_{dd} at a/d =2.0 show a significant difference (V_{yd}/V_{dd} =2.5), and V_{yd} is larger than V_{dd} at any a/d. This difference means that V_{yd} overestimates and/or V_{dd} underestimates the actual shear capacities if these have the continuity to a/d. The calculations show that the significant differences between V_{yd} (V_y) and V_{dd} (V_d) at a/d =2.0 occur when p_w and/or yield strength of shear reinforcement f_{wyd} is large.

In addition, the differences between V_{yd} and V_y are not the same as those between V_{dd} and V_d from Figure 3. The differences between the design equations and the experimental equations at the same a/d depend on the degree of reliability of the experimental equations. This is because the design equations V_{yd} and V_{dd} express the lower limits of V_{uexp}.

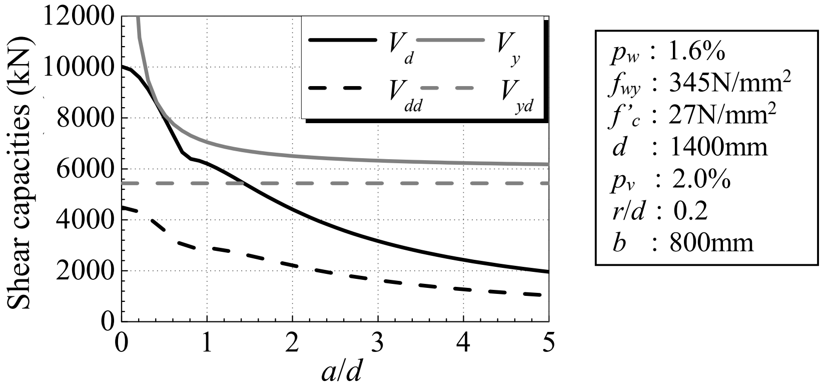

Figure 3. Continuity of calculated values for shear capacity to a/d

Research to date has used nonlinear finite element analysis to estimate the shear capacities for RC beams with large shear reinforcement. This research shows that the shear capacities of RC beams with vertical shear reinforcement are properly calculated using the upper limit of V_s from equation 5.

$$p_w \cdot f_{wyd} \;/\; f'_{cd} \leqq 0.1 \qquad (5)$$

Figure 4 shows comparisons of V_{uexp} and V_{yd} (=V_{cd} + V_{sd}) when equation 5 is considered. Moreover, according to research to date, the upper limit of f_{wyd} has been 25 f'_{cd} (f'_{cd}: design compressive strength of concrete). It is confirmed that V_{yd} contains all V_{uexp} when equation 5 is considered.

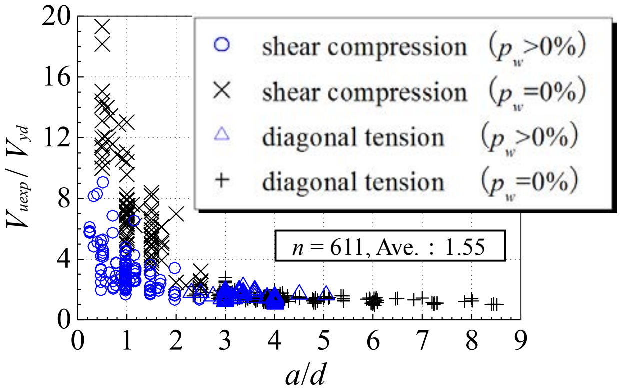

Figure 4. Comparison of V_{yd} (when equation 5 is considered) and V_{uexp}

Next, a method for considering r (r: length of bearing plate along the longitudinal axis) for V_{dd} is reevaluated. This is difficult to set up when calculating the shear capacities of actual structures. V_{dd} of equation 2 is condensed from V_d to r/d=0.05. Figure 5 shows a comparison of V_{dd_pw}, which is condensed to r/d=0.10, and V_{uexp} for RC beams with shear reinforcement. A comparison of the experimental results confirmed that V_{dd} contains all V_{uexp} for RC beams with shear reinforcement, even if r/d is 0.10. V_{dd_pw} for RC beams with shear reinforcement is calculated using equation 6.

$$V_{dd\_pw} = (\beta_d + \beta_w)\beta_p \cdot \beta_a \cdot \beta_r \cdot f_{dd} \cdot b_w \cdot d / \gamma_{bd} \quad (6a)$$

$$\beta_r = (1 + 3.33 \times 0.10)/(1 + 3.33 \times 0.05) = 1.14 \quad (6b)$$

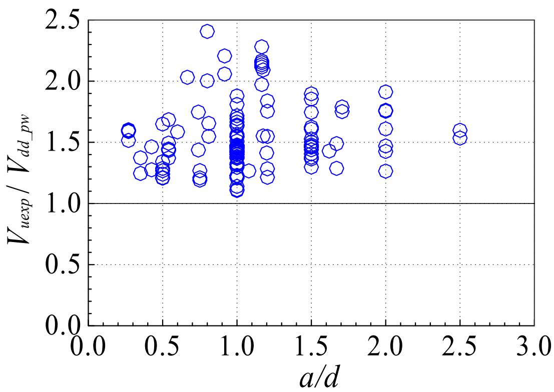

Figure 5. Comparison of V_{dd_pw} (r/d =0.10) and V_{uexp} (p_w>0%)

From the above results, the following equation is proposed as a method for calculating the design shear capacity to ensure continuity to a/d.

$$V_{design} = \begin{cases} \max(V_{yd},\quad V_{dd}) & \text{(without shear reinforcement)} \quad (7a) \\ \max(V_{yd},\quad V_{dd\_pw}) & \text{(with shear reinforcement)} \quad (7b) \end{cases}$$

V_{yd} is considered equation 5.

Figure 6 shows comparisons of V_{design} and V_{uexp}. It has been confirmed that V_{design} contain almost all of V_{uexp}. Figure 7 shows the relationships of the shear capacities and a/d for the same section in Figure 3. The proposed method can ensure continuity of the design shear capacity to a/d.

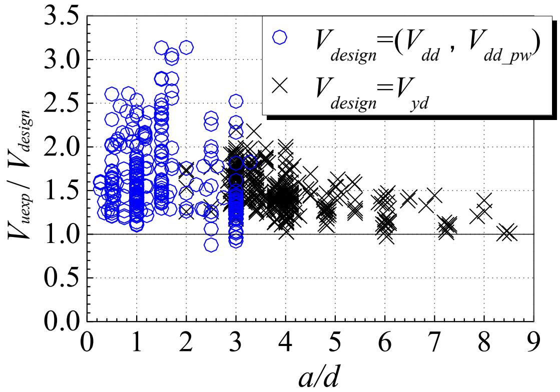

Figure 6. Comparison of V_{design} and V_{uexp}

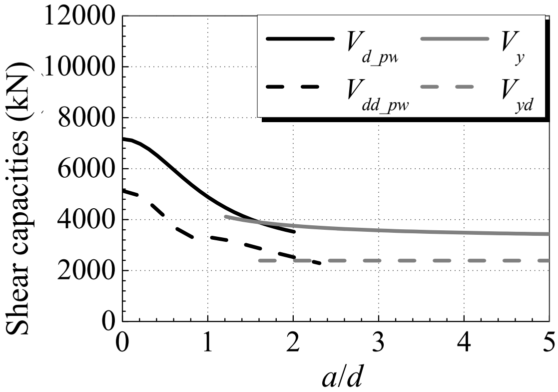

Figure 7. Continuity of calculated values of shear capacities to a/d using V_{design}

Reference

[1] Japan Society of Civil Engineers: Standard specifications for concrete structures - 2007 "Design," JSCE Guidelines for Concrete No. 15, 2010.12.